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# Reply to George Ch. Koumakis's Paper on Dialectic as Socratic Elenchus in Plato's *Gorgias*

ABSTRACT: This paper comments on George Ch. Koumakis's paper from the logical point of view; the author offers an explanation and puts forward the presupposition for transformation of the arguments into symbolic logic.

KEYWORDS: Sophists • dialectic • paradox • pragmatic and logical contradiction • analytic philosophy

## 1.

T here are different varieties of logic. The three which might seem most pertinent in this case are the propositional calculus (or sentential calculus), predicate logic (or quantification theory, presumably of the first order), and modal logic, since considerations of necessity, somehow – presumably logically – construed, might be involved. You formulate some arguments in the prepositional calculus. The first question here, and I do not think it is clear in Plato, is what logic, if any, of those at your disposal, would be appropriate as a translation of the Greek text, or, as I do not read Greek, an English translation of the Greek text. In short, we have the argumentation in a native language, not in a symbolic form. This raises the 'translation problem" - not the translation from Attic Greek to, say, modern Greek or contemporary English, but from the Attic Greek into a logic developed much later, and related to later languages and interests. Such a translation may be possible, but it is a problem (translation is always a problem). Confining oneself to a native language might be safer than trying to force the relationships and subtleties of a native language into perhaps overly simple, or perhaps reductionistic forms. Semantic entailments might not be governed by truth-functional limitations. For example, in the usual understanding of the prepositional

calculus, a legitimate substitution instance of a formula such as ' $p \supset q$ ' is taken to be true if and only if either the first proposition is false or the second proposition is true ('or' here is understood in the sense of inclusive disjunction, i.e., the disjunction is true if and only if either or both of the disjuncts are true; this is opposed to an exclusive disjunction which is true if and only if exactly one of the disjuncts is true). This is extremely counterintuitive. It also contradicts a number of linguistic intuitions, in such a way as to produce statements likely to be regarded by native speakers as simply absurd or clearly false. There are serious justifications for this procedure, which we need not enter into here, but it does raise the question as to whether or not the "logic" does justice to what it is supposedly representing.

Consider the following:

- 1) If Plato is a Greek, then Alexander is a Macedonian.
- 2) If Plato is a Greek, then Alexander is a Frenchman.
- 3) If Plato is a Frenchman, then Alexander is Macedonian.
- 4) If Plato is a Frenchman, then Alexander is a Frenchman.

In the prepositional calculus, as usually understood, statements 1, 3 and 4, proceeding, would count as true, and only statement 2 would count as false.  $\bigcirc$  is a truth-functional connective, not a semantic connective. If the four conditional statements above are normally construed, a native speaker would presumably take them either as nonsense, and thus as lacking truth-values, or as false, interpreting the 'if, then' relationship either as logical or nomological. For example, 'Plato is a Greek' does not logically entail that 'Alexander is a Macedonian'. Similarly, there is no law of nature to the effect that we know of, which nomologically links Plato's ethnicity with Alexander's ethnicity.

# 2.

There seem to be at least five different understandings of what might count as a paradox.

1) A paradox might be something which merely seems surprising or anomalous. For example, the Heisenberg Uncertainty Principle, to the effect that one cannot simultaneously establish both the location and velocity of a subatomic particle, might be regarded as paradoxical, or Schrödinger's Cat, which cat is supposedly neither dead nor alive until one looks, might be regarded as paradoxical. But these things are presumably neither contradictions in terms, nor logical contradictions, nor substitution instances of 'p.  $\sim$  p'. Similarly, if an individual who is usually very thoughtful should suddenly begin to behave in a rude manner, it might be regarded as unexpected, surprising or paradoxical, but no contradiction in a logical sense would be involved. 2) Consider a proposition such as ' $p \supset \sim p$ '. This is logically equivalent to ' $\sim p v \sim p$ ', which is logically equivalent to ' $\sim p$ '. Thus, it looks as though 'p' were entailing its own negation. To be sure, "material implication" is not an implication. What gives us ' $\sim p$ ' is not 'p', but the whole formula ' $p \supset \sim p$ '. Compare: "If Plato is a Macedonian", then Plato is not a Macedonian'. The whole formula here might be regarded as a paradox.

3) A more normal construal of a paradox, in the strong sense, would be a substitution instance of a formula resembling ' $p \equiv -p$ '; namely the one understood as 'p if and only if -p', and the 'if and only if' might be understood as the material biconditional (which is true if and only if both terms have the same truth value, either both true or both false), as a logical biconditional (each term logically entailing its own negation), or as a nomalogical biconditional (each term nomalogically requiring its own negation). In such a paradox, in the strong sense, each term involves the negation of the other.

A putative example of a logical paradox in the strong sense is: This statement is false. If the statement is true, it seems it must be false; and if it is false, since it says it is false, it must be true. To be sure, there are at least two other ways of handling this anomalous situation; first, the ordinarylanguage approach, and, second, the semantic approach, via linguistic levels. In the ordinary-language approach, the locution, out of context, is regarded as deviant, and not admissible.

A locution such as 'This statement is false' is acceptable in such a context as 'Plato is a Macedonian', followed by 'This statement is false', where the second statement is not self-referential, but clearly refers to the preceding statement. (It might be noted that self-referentiality is not always deviant of objectionable. For example, 'This statement is written in English' could be both self-referential and true.)

In the semantic approach, one distinguishes between levels of discourse, and one is permitted to refer to one level only in terms of another level, or a metalevel. For example, one refers to locutions only on the metalevel, and, similarly, predicates such as 'true' and 'false' are metalinguistic predicates and can occur appropriately only on a metalevel. For example, Plato is a Greek (Object-level discourse).

The statement 'Plato is a Greek' is true (metalevel discourse). Thus, 'This statement is false' would lack a truth value altogether. It would not be well formed. For example, "This statement" is false, would be unintelligible. For example, "this statement" is not a statement (in the objective language), and so it cannot have a truth value. Your problem, of course, is to try to figure out, and it may not be clear, what sort of paradox, if any, is involved in your concerns with Plato. It is my guess that the sort of paradox involved in Plato is not a logical paradox, or is not best construed as a logical paradox, but is, or is best construed as, a "pragmatic paradox". (This is not to deny that one might somehow work almost any anomaly into the form of a logical paradox, if one wished to do so. The problem, I suspect, is to come up with an interpretation which can most naturally be related to the text).

Pragmatic paradox. An excellent example of pragmatic paradox might 4) be taken to be the "Liar's Paradox", in which Epimenides, a Cretan, tells us that all Cretans are liars. Here we would not have a logical paradox. For example, the fact that all Cretans are liars, if true, does not imply that everything said by any Cretan at any time is a lie. For example, a good liar tells many truths, and thus sets his audience up for the important lie, which then will presumably be taken more seriously because it is uttered by a person noted for his truth telling. Good liars make a point of telling truths most of the time. Similarly, if a Macedonian said that all Cretans are liars, we would presumably regard him as mistaken, but not as guilty of a logical contradiction. The anomaly of Epimenides telling us that Cretans are liars is that Epimenides is himself a Cretan, but that is a contingent fact about Epimenides, the person, and not a proposition which enters into the logistic matrix. Another example of a pragmatic contradiction would be an individual who lights up a cigarette while telling us about the health hazards of smoking. There is no logical contradiction here, but his behavior seems to undercut his advice, or whatever. In a pragmatic contradiction we usually have some sort of discrepancy between words and actions, between presumed motivations and behaviors.

Note: One could make out a case that the Epimenidean case is not even a pragmatic contradiction, since the fellow is presumably kidding us, making a joke, entertaining the audience and so on, and does not intend to be taken seriously. If this is the case, i.e. he has no intention to be taken seriously, he may be perfectly consistent in what he intends doing; namely, perfectly consistent in his intent, say to shock, delight, confuse, befuddle and amuse.

It might be noted in passing that not all paradoxes are contradictions and not all contradictions are paradoxes. The first point we have mentioned already, but, to illustrate it again, the principle of compounding velocities fails where the speed of light is concerned. For example, the speed of light is constant regardless of whether one is in motion towards the source of light or away from the source of light. This is very paradoxical, but it is not a contradiction. It is, as far as we know, a fact of nature. With respect to the second point, most contradictions would not count as paradoxes. For example, 'Plato is a Greek and it is not the case that Plato is a Greek' is a contradiction, but it would not seem to be a paradox. A paradox usually involves something surprising or anomalous. For example, the assumption that every property determines a class, and that class membership is a property, and that there is a class of all classes which are not members of themselves will generate a contradiction, namely, that the class of all classes which are not members of themselves must both be a member of itself and cannot be a member of itself. (Russell's Paradox). There are, as you know, many paradoxes. Several are mentioned, for example, in William and Martha Kneale's *The Development of Logic* (Oxford, at the Clarendon Press, first published in 1962).

5) I am listing herewith a fifth sort of situation, which might be regarded as a paradox. This is the situation where one has an implicitly contradictory premise set, which seems anomalous, and then one derives an explicit contradiction from the premise set. This may be the closest thing to what you may have in mind. To be sure, whether or not one calls this a paradox would seem to be a matter of interest or choice. For example, a contradictory premise-set, in itself, is not likely to be regarded as paradoxical. On the other hand, one discovers that it is contradictory when one sees that it entails a contradiction, and one finds this charming, or the contradiction seems unexpected or interesting. I discuss this sort of thing later in these notes.

At this point, particularly in connection with this "fifth sort" of situation, I think some observations are in order. In the sort of situation in which you appear to be interested, there seem to be three phases involved:

- Phase 1: The phase of "pragmatic paradox", where, say, the Sophist's actions are discovered to be self-frustrating, self-defeating, or such. For example, he teaches a student to be dishonest and then is victimized by the dishonesty of the student, who refuses to pay his fees, or something like that. There is no logical contradiction here, as logical contradictions require truth-value-bearing entities, say, propositions or statements.
- Phase 2: Socrates, or someone, notes, or thinks he notes, a situation which can be propositionalized in such a way as to construct a premise-set, which will entail an explicit contradiction.
- Phase 3: Socrates, or someone, constructs an argument, presumably a sloppy argument, or an incomplete argument, which he then attributes, probably mistakenly, to the Sophist.

I will illustrate how this might proceed with an invented example:

- Phase 1: Individual, wishes to be healthy but continues to consume large amounts of alcohol and smoke heavily.
- Phase 2: Someone notes that Individual, wishes to be healthy but yet continues to consume large amounts of alcohol and smoke heavily, and this someone realizes that there is a self-restorative behaviour involved, a conflict between a supposed motivation and a form of activity. There is some sort of the conjunction, implicit or explicit, of truth-value-bearing entries, say, propositions or statements.
- Phase 3: Someone decides to propositionalize this situation in such a way as to construct a contradictory premise-set, from which an explicit contradiction may be derived.

He might do so along the following lines:

- 1. Individual, wishes to be healthy (premise).
- 2. Individual consumes large amounts of alcohol and smokes heavily (premise).
- 3. If Individual, consumes large amounts of alcohol and smokes heavily, then it is not the case that Individual, wishes to be healthy (premise).
- 4. It is not the case that Individual<sub>1</sub> wishes to be healthy. 2, 3, *modus ponens*.
- 5. Individual, wishes to be healthy and it is not the case that Individual, wishes to be healthy, 1, 4, conjunction.

The argument could be formulated more plausibly by using the predicate calculus, but it can be formulated with fewer steps in the prepositional calculus, as above. The form then, in the prepositional calculus, would be:

1. p	(premise)
2. q	(premise)
3. $q \supset \sim p$	(premise)
4. ~ p	(2, 3, modus ponens)
5. p. ~ p	(1, 4, conjunctions)

Note: The third premise, above, is necessary to generate the conclusion, but it could certainly be regarded as false. Then the argument, while valid, would be unsound (an argument is sound if and only if it is valid and has a true premise-set). Indeed, Individual, wants to have it both ways. To make the argument plausible, rather than merely valid, one would seem to require additional premises, perhaps to the point of involving ourselves with psychological theory, dividing motivations into tiers, conscious and subconscious,

considerations pertaining to self-destructive tendencies, *etc.* In short, an argument which seems simple may not really be simple at all. For example, in your context, Socrates is simply assuming that the nonpayment of fees is wicked, or such. Technically, if one is going to have a complete argument, and one that is also plausible or interesting, it seems these suppressed premises, and there may well be several of them, should be spelled out to be made explicit. For example, suppose the student believes that, in certain instances, the nonpayment of fees is morally obligatory. He might think, for instance, that it is immoral to pay an instructor for immoral teaching, and his duty is to discourage such people from teaching antisocial subject matters. He might also have a theory that it is morally acceptable not to pay individuals who have red hair or are wearing purple himations, and the Sophist has red hair and is wearing a purple himation.

I suspect that some, or perhaps all of the "contradictions" in Platonic text here might be best construed as "pragmatic contradictions", and not as logical contradictions. Another concept of the pragmatic contradiction is that it is a self-frustrating form of behavior and, if we interpret it that broadly, propositions, statements and such need not even enter into a pragmatic contradiction. For example, let us suppose we have an individual who thinks that he is silly, doubtless correctly, but also wants to be successful in that culture. Then, let us suppose we have an individual who thinks that he is silly, doubtless correctly, but also wants to be successful in that culture, so he does not wear a yellow hat. He is not successful, of course, because he is looked down on, and discriminated against, for not wearing a yellow hat. He may be a noble, sensible fellow, but he is also guilty of pragmatic contradiction. For example, he should either wear a yellow hat or give up the desire to be successful in that culture. (He may, of course, wage a campaign against yellow hats, or something like that but, in such a case, he is not guilty of pragmatic contradiction because he recognizes that he cannot, at present, achieve success in the culture without a yellow hat. Therefore he has to change the culture first. Another example of pragmatic contradiction without propositions being explicitly involved is the fellow who wants to eat his cake and have it, too. Many people live in pragmatic contradiction, for example a woman who wants to be both slender and yet eats like a hippopotamus, a fellow who recognizes the health hazards of alcohol and yet drinks excessively, and so on.)

Let us consider two cases which are closer to the Platonic concerns:

Case 1: We have a Sophist, let us call him Sophist, who successfully teaches a pupil how to be clever, get ahead, *etc.* He shows the pupil, for example, how to be a "winner" in an immoral, ma-

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terialistic, competitive society, by emphasizing prudential selfinterest, even at the expense of moral propriety, the welfare of others, *etc*.

Case 2: We have a Sophist, let us call him Sophist<sub>2</sub>, who claims to teach a pupil to be good, i.e., to be honest, pay his debts, *etc*.

Naturally, in both cases the teacher does not yet get paid. Socrates seems to think, incidentally, that there is something immoral in taking money for teaching people to be good. He does not prove that, as far as I know. (Are we to gather that teachers in day-care centers, in nursery schools, in kinder-gartens, *etc.*, i.e. the ones whose main job is to socialize the current crop of little invading barbarians, should not be paid?) Also, he seems to think that the teacher of goodness (and he seems to implicitly recognize, contrary to the hypothesis, that goodness can be taught) will be rewarded in kind, and thus, in a sense, paid, because the pupil will be good, the gifts of virtue will be exchanged, the teacher will be appreciated, esteemed, treated well, *etc.* 

In the first case, I think we have a situation which can be construed as one of pragmatic contradiction. In the second case, I think we have a situation where the teacher is simply mistaken. In short, he failed to teach the pupil to be good. I do not see a pragmatic contradiction in the second case. We can infer, of course, that the teacher did not manage to teach the pupil how to be good. It does not follow, of course, that the failure of one teacher to impart virtue proves that virtue cannot be taught. Deductively the inference is invalid. It does, of course, afford some inductive evidence for the nonteachability of virtue. On the other hand, the overwhelming amount of evidence is on the other side, namely that virtue can be taught. It is socialization; it is taught by parents in particular, but also by other adults, siblings, peers, life experiences, *etc.* To take virtue as an inexplicable gift of the gods or the result of recollecting forms in another existence in certainly not the most plausible hypothesis here.

Let's consider the first case:

Sophist intends to tutor his charge in sharp or corrupt practices. He, being a man of the world and so on, should then recognize that he is leaving himself open to be the victim of such practices on the part of the student. An analogy would be that if a fellow teaches someone how to pick pockets, he must risk having his own pocket picked by that person. Assuming that he does not wish to have his pocket picked, he should realize the danger to himself in imparting these antisocial skills, and refuse to impart those skills, unless, say, he is paid far more than he is likely to lose by having his pocket picked. There are two differing possibilities here: first, if the teacher knows he will not be paid, will be betrayed, *etc.* and objects to these things, he is clearly guilty of pragmatic contradiction. For example, a fellow who does not wish to be burned and yet thrusts his hand into a fire is guilty of pragmatic contradiction. But presumably the teacher does not know he will not be paid. This brings us to the other possibility. Should he have anticipated this development? It seems he should have recognized its likelihood, if not its inevitability. In such a case I think we could also assign pragmatic contradiction. Similarly, a fellow who realizes the danger of being burned should not thrust his hand into the flame. Probabilities are involved here, however. For example, if the teacher thinks there is a 95% probability, or such, that he will be paid, it might be rational, if not moral, to instruct his pupil. It is not clear, of course, that pragmatic contradiction is involved if the teacher is simply stupid or uninformed. Similarly, if a child does not know that putting his finger into the flame will hurt, then the child, though ignorant, is not guilty of pragmatic contradiction. In any event, it seems the teacher we have called Sophist, would be well advised to collect his fee in advance. And if he does not get his fee, it is hard to feel sorry for him. One might suppose that it served him right. Poetic justice, etc

Let us consider the second case.

Here Sophist, is well-intentioned, wants to teach the pupil to be good, *etc*. On the other hand, the pupil does not pay him for the instruction. I do not see any contradiction here, pragmatic or otherwise. We may certainly infer that the claim of the teacher to teach goodness is counter-exampled in at least this single case. This does not mean, of course, that he might not be successful in his other instructions, with his other pupils, *etc*. (Indeed, considering the success, wealth, *etc*, of many Sophists we may suppose that their instruction paid off, had value, *etc*. If it was prized in the free, open, unconstrained, competitive market, it presumably satisfied needs, demands, *etc*.) Also, it seems unfair to lay a student's failure on the doorstep of the teacher, at least inevitably. The teacher can teach, but he cannot learn for the student. Learning is the student's responsibility. Similarly, a musician can play music but he cannot hear it for the audience. They have to listen, and hear it for themselves.

As noted, there are several senses of the word 'paradox'. It seems to me that if paradoxes are involved in sophistic teaching, they would either be paradoxes in the innocent sense of being surprising or unexpected developments, say, not being paid for services rendered, or paradoxes in the sense of "pragmatic paradoxes", for example engaging in a behavior which is self-frustrating, self-defeating or such. A Sophist of the Sophist<sub>1</sub> sort might be involved in a paradox of the first sort; a Sophist of the Sophist<sub>2</sub> sort might possibly be involved in a paradox of the second sort. It does not seem that either Sophist is involved in a paradox of the number -2 or number -3 sort earlier alluded to.

Some sort of case might be made out for a paradox of the number -2 sort, the '~ p  $\supset$  p' sort, but I think that would be str*etc*hing things. It might go like this:

If a Sophist teaches virtue, then it is false that a Sophist teaches virtue.

This proposition is logically equivalent to 'It is false that a Sophist teaches virtue', yet the statement does not follow from the first term, but only from the conditional as a whole, and the conditional as a whole really amounts to no more than the assertion that it is false that a Sophist teaches virtue, which seems to be merely making an assertion, not providing an argument. It may look like some sort of argument, but it seems more like simply asserting something, and even like the fallacy of begging the question.

The full-blown, strong-sense, logical paradox, e.g., a paradox of the ' $p \equiv \sim p$ ' sort, the 'p if and only if not-p' sort, does not seem to be involved here. In short, I do not think a paradox of the Barber sort, or of the Russell sort, is involved here. I suspect the texts involved would not consent to such an interpretation, or recasting. It is an independent question whether or not such an effort, with sufficient labour and ingenuity, might be made plausible. I suspect that would ultimately be injudicious, and probably a mistake.

If we are dealing with entailment issues and not material implications, or with semantic issues and not truth-functional issues, one might, to be fairer to the text, need a different logic. I am not sure of that, however. A good deal would seem to depend on whether or not one needs single-proposition entailments. For example, '*a* is a circle' logically entails '*a* is a geometrical figure', but the analyticity of the conditional 'If *a* is a circle, then *a* is a geometrical figure' would not be clear if it were formalized as 'C  $\supset$  G'. Again, I think your best bet might be to avoid possible pitfalls of risky "translation" into a symbolism, and to confine yourself to natural-language argumentation. To be sure, this probably depends on the argument, the context, *etc*.

One could, of course, construct arguments which would entail contradictions.

For example:

1. The Sophist teaches virtue.

2. If the Sophist teaches virtue, then his pupil is virtuous.

- 3. It is false that his pupil is virtuous.
- 4. It is false that the Sophist teaches virtue. 2, 3, *modus tollens*.

5. The Sophist teaches virtue and it is false that the Sophist teaches virtue. 1, 4, conjunction.

This has the form:

1. p	(premise)
2. p⊃q	(premise)
3. ~ q	(premise)
4. ~ p	2, 3, Modus Tollens
5. p. ~ p	1, 4, Conjunction

One could also derive an infinite number of contradictions from this premise-set, as it is inconsistent. Another contradiction would be derived as follows:

- 1. The Sophist teaches virtue.
- 2. If the Sophist teaches virtue, then his pupil is virtuous.
- 3. It is false that the pupil is virtuous.
- 4. His pupil is virtuous. 1, 2, modus ponens
- 5. His pupil is virtuous and his pupil is not virtuous. 4, 3, conjunction has the form:

This has the form:

1. p	(premise)
2. $p \supset q$	(premise)
3. ~ q	(premise)
4. ~ q	1, 2, Modus Tollens
5. q.~q	4, 3, Conjunction

From a contradiction, of course, all propositions, whether or not contradictory in themselves, may be derived.

For example:

1. p. ~ p	
2. p	1, Simplification
3. ~ p. p	1, Commutation
4. ~ p	3, Simplification
5. pvq	2, Logical Addition
6. q	4, 5, Disjunctive Syllogism

And here 'q' could be any proposition whatsoever, whether contingent, contradictory, or analytic. (To be sure, an analytic proposition, one true in virtue of its meaning alone, can be derived form any proposition whatsoever, whether contingent, contradictory, or analytic.) So it is not hard to generate contradictions from these materials. Perhaps this is the sort of thing you are looking for. This is a far cry thing, like Russell's Paradox, but I suppose you might count it as some sort of paradox. I myself am not enthusiastic about taking every contradictory premise-set as a paradox, but I suppose one could take some of them as paradoxes.

For example:

*S* is a swan and it is false that *S* is a swan, does not seem like a paradox. Similarly:

- 1. S is a swan
- 2. If S is a swan, then Plato is a Greek
- 3. If Plato is a Greek, then Alexander is a Macedonian
- 4. If Alexander is a Macedonian, then it is false that S is a swan,

is a contradictory premise-set, but it does not seem the sort of thing one would call a paradox.

I am not sure that recourse to symbolism in Platonic cases represents much of an advance over a careful restatement of the original texts themselves. One technique, however, is to propose "models" of arguments. You can control the logic of a model, and it can be very helpful, if fair to the original text. In short, the original text may be redundant, confused, and obscure, but sometimes you can construct a "model" of a contained, implicit, or neighbouring argument, and thus shed light on at least the gist or drift of the original argument. Perhaps that is what you would like to do. If you do want to claim logical paradoxes in cases of this sort, you should make certain that the premise-set is inconsistent, and then derive at least one explicit contradiction from the premise-set. For example, there is nothing paradoxical, in anyone's book, about an instance of a valid argument form, such as, say, modus tollens. Incidentally, Socrates, does not sometimes seem very fair to me. For example, in asserting things like: "you have made him good and he still is; yet he remains wicked" (Italics mine). Obviously no one, including any Sophist, is going to maintain that his pupil is both good and not good at the same time, in the same respect, etc. Similarly, the whole Socratic oddity of forms seems unilluminating. For example, acts of injustice are due to injustice. This does not seem even as illuminating as the explanation that a soporific powder's capacity to induce sleep is due to the powder's sleep-inducing capacity (Cf. Moliére). Are acts of carpentry due to carpentry, acts of motoring due to motoring, acts of kicking due to kicking, etc? And how would injustice act in the world? How would it bring about an act of injustice? How could it do work, whence the energy, how the effect, etc.?

A proposition such as "'A' is just if and only if it is not the case that 'A' is just" is an outright contradiction.

For example:  $p \equiv \sim p$ 

is logically equivalent to:  $(p \supset \sim p) \cdot (\sim p \supset p)'$ , which is logically equivalent to  $(\sim p \lor \sim p) \cdot (\sim \sim p \lor p)'$ , which is logically equivalent to  $(\sim p \lor p \lor p)$ , which is logically equivalent to '~ p . (p v p )', which is logically equivalent to '~ p . p', which is logically equivalent to 'p . ~ p'.

Thus, it would be logically equivalent to 'A is just and it is not the case that A is just', a substitution instance of the contradictory from 'p .  $\sim$  p'. But is it fair to impose such an absurdity on the Sophists? It does not seem so to me. If the pupil turns out to lack virtue, obviously the teaching has failed, and he cannot simultaneously be regarded as having virtue. It doesn't seem you can have it both ways, even if you are Socrates. I wonder if Aristotle, who had a good head for logic, ever pointed out this sort of thing to Plato. Or perhaps Plato just wants us to think about these things, and his job is more to get us to do philosophy, rather than to convert us to a particular set of philosophical propositions. (My colleague, Professor Henry G. Wolf, now retired, had some interesting ideas along those lines, seeing Plato, I think, largely in terms of stimulatory, intellectual drama, an individual out to challenge and test the reader, and excite him, and stimulate him, to see through and attack a variety of positions. In short, as I understand Dr. Wolz, a good deal of the faulty, and sometimes preposterous logic in the dialogues was intentional, part of an ingenious plot to provoke the stimulating and redemptive activity of philosophical dialectic. An interesting book of his is *Plato and Heidegger*: In Search of Selfhood.)

In passing, since you speak of disjunction, dilemmas and such, I might mention that there are two generally recognized types of dilemma, i.e. constructive and destructive.

The argument forms are as follows:

Constructive Dilemma:  $(p \supset q) \cdot (r \supset s)$   $p \lor r$   $q \lor s$ Destructive Dilemma:  $(p \supset q) \cdot (r \supset s)$   $\sim q \lor \sim s$  $\sim p \lor \sim r$ 

It does not follow, of course, that a subject is not teachable if teachers do not exist. A subject is teachable if it can be taught. This does not require the existence of actual teachers. For example, archery is the sort of thing that can be taught, even if no one is teaching it these days. Similarly, archery is teachable even if no one is a student of archery these days. Let us suppose there is some secret of stained-glass windows, which was lost in the Middle Ages. Presumably the secret is such that it could be taught, even if it is no longer taught. Similarly, if the secret is rediscovered, or shows up in some newly discovered manuscript, one could start teaching the technique again.

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I have not, at least on the whole, tried to draft particular arguments pertinent to the various points, which you reference in your notes. I think, on the other hand, that I have supplied enough notation here and, more importantly, enough background considerations, to give you some of the sort of approach which analytic philosophy or, better still, an analytic philosopher, might take with respect to these issues. I hope I have been of some help. On the other hand, I suspect that a certain amount of risk is involved here, in attempting to formalize arguments on the basis of texts, in which a number of premises may be suppressed or taken for granted. I will set forth one argument, in the propositional calculus, as a sort of gesture in the direction of formalization.

N: Virtue is acquired by nature

T: Virtue is acquired by teaching

D: Virtue is acquired by divine dispensation

Plato's argument seems very poor to me, seeming to rely on a premiseset which seems either false or nonsensical, or both. For example, it seems obvious to me that virtue can be taught, because we see it being taught all about us, everywhere, for centuries. Similarly, the notion of a "divine dispensation" seems cognitively unintelligible, or simply nonsensical to me. What is the divine, which divine, how would it work, what is it like, *etc*. Also, if virtue is a gift from some mysterious source, there would seem to be little point in striving to attain it. To be sure, I suppose one could always petition that it would be granted to one, but which god or gods would one ask? Any one of them, all of them? The virtue of Ares might not be the virtue of Aphrodite, *etc*.

1.	N v (T v D)	(premise)
2.	~ N	(premise)
3.	~ T	(premise)
4.	T v D	1, 2, Disjunctive Syllogism
5.	D	4, 3, Disjunctive Syllogism

The following argument is valid:

1.	(∃x) Px	
2.	(x) $(Px \supset Jx)$	
3.	$(\mathbf{x}) [(\mathbf{P}\mathbf{x}  .  \mathbf{J}\mathbf{x}) \equiv (\mathbf{P}\mathbf{x}  . \sim \mathbf{J}\mathbf{x})$	z)]
4.	Ру	1, Existential Instantiation
5.	$Py \supset Jy$	2, Universal Instantiation
6.	Jy	4, 5, Modus Ponens
7.	Py.Jy	4, 6, Conjunction
8.	$(Py . Jy) \equiv (Py . \sim Jy)$	3, Universal Instantiation

9. $[(Py . Jy) \supset (Py . \sim Jy)]$ .	[( Py . ~ Jy) $\supset$ (Py . Jy)] 8, Equivalence
10. (Py . Jy) ⊃ (Py . ~ Jy)	9, Simplification
11. Py.~Jy	7, 10, Modus Ponens
12. ~ Jy . Py	11, Commutation
13. ~ Jy	12, Simplification
14. Jy . ~ Jy	6, 13, Conjunction
15. (∃x) (Jx . ~ Jx)	14, Existential Generalization

Line 15 is a contradiction. This may be demonstrated by the following argument:

1.	(∃x) (Jx . ~Jx)	
2.	Jy.~Jy	1, Existential Instantiation
3.	~Jy . Jy	2, Commutation
4.	Ју	2, Simplification
5.	~Jy	3, Simplification
6.	Jy v (p . ~ p)	4, Addition
7.	p. ~ p	5, 6, Disjunctive Syllogism
e ab	ove arguments:	

In the

Px:	<b>x</b> is a pupil of a Sophist
Jx:	<b>x</b> is just to himself
~Jx:	<b>x</b> is not just to himself

The conclusion of the first argument above, of course, is that there exists at least one entity which is just to himself and not just to himself. It might be noted that line 3 above, toward the top of the page, is not a contradiction. For example, consider the following argument:

1.	$(\mathbf{x}) [(\mathbf{P}\mathbf{x} . \mathbf{J}\mathbf{x}) \equiv (\mathbf{P}\mathbf{x} . \sim \mathbf{J}\mathbf{x})]$	
2.	$(Py . Jy) \equiv (Py . \sim Jy)$	1, Universal Instantiation
3.	$(\exists x) [(Px . Jx) \equiv (Px . ~Jx)]$	2, Existential Generalization

Neither line 1, directly above, nor line 3, just above, is a contradiction. For example, the conclusion, line 3, just above, could be true, and would be true if nothing had the property of being a student of a Sophist. As an analogy, consider the following formula in the prepositional calculus:  $(p \cdot q) \equiv (p \cdot q)$ 

This formula would be false if 'p' were true, but if 'p' were false, it would be true. A biconditional formula is true if both sides have the same truth value, e.g. both sides are true or both sides are false. Things would be different if, say, we had a formula like '(Ex) ( $Px \equiv \sim Px$ )'. For example, then:

1. 
$$(\exists x) (Px \equiv \sim Px)$$

2. 
$$Py \equiv \sim Py$$

2.  $Py \equiv \sim Py$ Existential Instantiation3.  $(Py \supset \sim Py) \cdot (\sim Py \supset Py)$ 2, Equivalence

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4. (~ Py v ~ Py). (~ ~ Py v Py)	3, Implication
5. $(\sim Py v \sim Py) . (Py v Py)$	4, Double Negation
$6.  \sim \mathbf{P}\mathbf{y} \cdot \mathbf{P}\mathbf{y}$	5, Tautology
7. Py.~Py	6, Commutation
8. $(\exists x) (Px . ~ Px)$	7, Existential Generalization
8. Py	7, Simplification
9. ~ Py . Py	7, Communication
10. ~ Py	9, Simplification
11. Py v (p . ~ p)	8, Addition
12. p. ~ p	10,11, Disjunctive Syllogism

One would commonly telescope some of these steps, but I have chosen, for better or for worse, to make them explicit. I hope the above arguments are intelligible. I have used "~" for negation; "." for conjunction; "v" for inclusive disjunction; " $\supset$ " for the material conditional (if, then); and " $\equiv$ " for the material biconditional (if and only if). It must be clearly kept in mind that the material conditional and the material biconditional are truth-functional connectives, not semantic ones. For example, a material conditional is true if and only if either the antecedent is false or the consequent is true; and a material biconditional is true if and only if both terms have the same truth value, either both true or both false.

For ' $(\exists x)$ ( Px') to be true, it is necessary that there is at least one entity which has the property "P." For '(x) Px' to be true, it is necessary that all the entities have the property "P." The first formula is presumably weaker, and the second formula is presumably much stronger than you would like them to be.

'(x)  $[(Px \supset Jx) . (Px \supset ~Jx)]$ ' does not imply that anyone has the property "P." Indeed, it logically implies that no one has the property "P." For example, this formula is logically equivalent to '(x)  $(Px \supset (Jx . ~Jx)$ ', which is logically equivalent to '(x) ~ Px', e.g. everything is not "P," or nothing is "P". For example:

1.	$(\mathbf{x}) [\mathbf{P}\mathbf{x} \supset (\mathbf{J}\mathbf{x} . \sim \mathbf{J}\mathbf{x})]$	
2.	$\mathbf{P}\mathbf{y} \supset (\mathbf{J}\mathbf{y}  . \sim \mathbf{J}\mathbf{y})$	1, Universal Instantiation
3.	~ Pv v (Iv . ~ Iv)	2. Implication

Line 3 here is logically equivalent to '~ Py'. Consider the following: p p v (q . ~q)

These two formulas are interdeducible, i.e. each of them logically implies the other. If the numerator formula is true, the denominator formula has a true disjunct and thus is true. If the denominator formula is true, it has to be true because 'p' is true, as a contradiction is always false. Accordingly,

Or:

if the denominator formula is true, the numerator formula must be true. Thus the two formulas are interdeducible, each of them implying the other. To continue, one may derive the nonexistence of anyone with property "P" from the formula, as follows:

3.	~ Py v (Jy . ~ Jy)	2, Implication
4.	(~ Py v Jy) . (~ Py v ~ Jy)	3, Distribution
5.	$(Py \supset Jy) . (\sim \sim Py \supset \sim Jy)$	4, Implication
6.	$(Py \supset Jy) . (Py \supset \sim Jy)$	5, Double Negation
7.	$(\mathbf{P}\mathbf{y} \supset \mathbf{J}\mathbf{y}) \ . \ (\sim \sim \mathbf{J}\mathbf{y} \supset \sim \mathbf{P}\mathbf{y})$	6, Contraposition
8.	$(Py \supset Jy) \cdot (Jy \supset \sim Py)$	7, Double Negation
9.	$Py \supset \sim Py$	8, Hypotherical Syllogism
10.	$\sim$ Py v $\sim$ Py	9, Implication
11	~ Py	10, Tautology
12.	$(\mathbf{x}) \sim \mathbf{P}\mathbf{x}$	11, Universal Generalization

I hope that all this is of some help. Some of your arguments have the following forms:

1.	$A \supset (B . C)$	
2.	~ B (not B)	
3.	$\sim B v \sim C$	2, Logical Addition (not-B or not-C)
4.	~ (B . C)	3, DeMorgan Transformation
5.	~ A	1,4, Modus Tollens

You have an argument which might be set out as:  $(A \lor B) \supset C$ 

A C

~ p

This is a valid argument. For example:

1.	$(A \lor B) \supset C$	
2.	А	
3.	A v B	2, logical addition
4.	С	3, 1, modus ponens

An argument is invalid, as you know, if and only if it is logically possible for the premise-set to be true and for the conclusion to be false. An argument is valid if and only if it is logically impossible for the premise-set to be true and for the conclusion to be false. Interestingly, it follows from this criterion for validity that a contradiction implies all conclusions, and that any analytic conclusion (logically a true conclusion) is implied by all premise-sets.

I gather that another argument is to be read:  $p \supset \sim q$ q This is a valid substitution instance of *modus tollens*. Some purists might want you to have it as follows:

1.	$p \supset \sim q$	
2.	q	
3.	~ ~ q	2, Double Negation
4.	~ p	3,1, Modus Tollens

but that seems a bit pedantic to me. To be sure, some folks might worry about it.

Two propositions are materially equivalent if and only if they have the same truth value. This is very different from logical equivalence. For example: 'Berlin is the capital of Germany' and 'Athens is the capital of Greece' are materially equivalent. Similarly, 'Berlin is in Germany' and 'Athens is in Greece' are materially equivalent, but not logically equivalent.

Two statements are logically equivalent if and only if they must, of logical necessity, have the same truth value. Alternatively, two statements are logically equivalent if and only if they are interdeducible. It might also be noted that logical equivalence and synonymy are different relations. All synonymous statements are logically equivalent, but not all logically equivalent statements are synonymous. For example:

'p' is logically equivalent to 'p v  $(p \cdot q)$ ' but they would not have the same meaning. All logically true statements, incidentally, are logically equivalent, and all inconsistent statements are logically equivalent, but obviously they would not have to have the same meaning, i.e. would not have to be synonymous. For example:

substitution instances of 'p  $\supset$  p' and 'q v ~ q' would be logically equivalent, and substitution instances of 'p . ~ p' and 'q . ~ q' would be logically equivalent, but synonymy would not have to be involved.

Some other remarks:

Let us suppose that virtue is really the result of either (1) "divine fate" or (2) "divine grace".

(1) The "divine fate" approach suggests determinism, for better of for worse. Alcibialdes was fated to betray Athens, and so on. He couldn't help it. And I am not sure that the day is saved by supposing that, as in the myth of Er, folks get to freely choose their lives. If that was true, why would so many folks choose to be short lived of diseased, to be lowly laborers, to live in poverty and so on? Is their choice free? Why would it not be fated as much as the choices in terrestrial life, *etc*? If "divine fate" is involved anywhere, it seems it should be involved anywhere from side to side and from top to bottom, so to speak.

(2) The "divine grace" approach doesn't seem to be of much help either. The notion of "divine grace" seem unPlatonic in the first place, as it seems to invoke the notion of a personal divine entity who manages the world and people. Plato's demiurge seems amoral, and is presumably impersonal. And the form of the Good, as far as I know, is not a artificial entity in any personal sense. Presumably it is, in its glory, placidly and sublimely indifferent to the "realm of becoming", to the "pollutions of mortality" and such. Also, if it weren't, then one would find oneself involved, as in Christian theology, with questions of predestination and so on, the divine entity provisioning reality, and organizing it in virtue of its decisions to dispense its "grace", favors, or such. Plato's notion about blaming teachers for the failures of students sounds like something coming out of a department of education, and is no more plausible. Suppose one would try to teach Jones to play the violin. If Jones fails to master the violin, either through indolence, unwillingness to practice, simple lack of talent, or whatever, this does not mean that the teacher can't teach the violin. One's proficiency at teaching the violin, morality or anything else should not be impugned by occasional failures on the part of students. One "can't win them all". As a teacher you are certainly well aware that reciprocity is involved in the pedagogical relationship.

Plato's notion of the unteachability of virtue is counterexampled by each generation. Virtue, morality, *etc.* are taught and, on the whole, successfully, since otherwise we would live in a worse jungle than we do. Further, it is my impression, for example, to her (ie, virtue) traits are now more likely because of given instruction and example, than re-collecting the form of virtue from some experience gained of supranatural existence.

I do not think Plato is always fair to his opponents. It seems to me that they must have often been competent, moral, well-intentioned, intelligent people. The paradoxes in which he seems determined to involve them often seem labored, strained and a bit silly. In the actual dialogue, where Plato is not pulling the strings, it seems they would have responded to him simply and clearly, and probably successfully. He seems more concerned to destroy them than to understand them. There is an expression in English to the effect that one might "demonize" an opponent; namely present him in such a way as to suggest that he is corrupt, witched, unprincipled, dishonest and so on. It seems to me that Plato has a tendency, on the other hand, to "clownize" his opponents, which word I have invented, but which suggests that he is concerned to make them seem like clowns, buffoons or fools. Whereas this may have some propaganda value in culture wars, it seems distinctly short on philosophical charity, or even philosophical civility.

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The question of time might enter into some of these matters. For example, one might be just at one point and unjust at another. This is obvious, but it might also have some application to your concerns. For example, if Jones is just at time  $t_1$ , and unjust at time  $t_2$ , there is no contradiction involved, no more that the fact that you, say, were twenty years old at one time and thirty years old at another time.

By definition nothing corresponding to a contradiction can exist, in the same respect, at the same time and so on. For example, it is logically impossible to be red and not red, at the same place, at the same time, in the same respect and so on. Similarly, it would be logically impossible for someone to be just and not just at the same time, in the same respect and so on. Thus, the most Plato can do is to attempt to show that the claims of the Sophists might be inconsistent, self-defeating, or so on. And to do that, it seems he would have to be a great deal more explicit about the nature of justice, etc. than he is. The theory of forms seems almost to be an act of faith, driven by the desire to justify objectivity claims, possibly particularly in moral matters. Hopefully, one can justify substantial objectivity in moral matters without recourse to nonaturalistic hypotheses of a peculiar and unconformable nature. Intuition seems to be the methodology, but the resolution of intuitional conflict is notoriously problematic. Platonic metaphysics seems to be an unreliable and tenuous ground for epistemology. (Many folks, of course, regard Plato's arguments for the existence of forms as inconclusive, at best. Some might even regard them as transparent rationalizations concocted to support a hypothesis to which one is independently and previously committed, perhaps more on religious or psychological grounds than on rational grounds).

There is one thing I find hard to follow: your identification of dialectic (or a version of it) with the "idea of Good" which, I take it, is the Form of the Good. I would have supposed that dialectic (in an advanced form) is the m e a n s whereby one attains the vision of, or understanding of the Form of the Good, and would not itself b e the Form of the Good. This seems a bit like conflating the airplane with the airport, or the railroad with the depot, namely the vehicle used to get somewhere with the destination desired. But then I may have misunderstood you here. Philosophers are good at misunderstanding one another. It is one of the things they do best. The "Ithaca poem" in your earlier work is a wonderful poem, even in translation. Doubtless it is even better in Greek. In this sense, the end might be the means, and the means the end, meaning that the journey is what counts, not just arriving at a particular destination. Perhaps that is what you had in mind with the "identification" of dialectic with the "idea of Good". But, even if it is true that the journey is the "end" of the *bonum*, so to speak, it would be an independent question whether Plato identified a version of dialectic with the Form of the Good. This seems to me unlikely. For example, dialectic presumably requires a rational mind to do it, rather like carpentry (doing carpentry) requires carpenters, whereas the Form of the Good is presumably independent, eternal, sublime, venerable, changeless and so on.

Consequently, it seems very strange to think of identifying good with dialectic (or knowledge). Dialectic presumably has to do with reasoning, thinking, arguing, seeking and such. It is presumably an activity. On the other hand, Good, as a target or a goal, is presumably not an activity but, in Platonic terms, an eternal imperishable, changeless, ideal form. It does not depend, presumably, on human activity. Similarly, whereas knowledge may be a good, it would seem incorrect to identify it with the good. Presumably, knowledge requires an object, and the knowledge is not identical with the object. Knowledge is mind-dependent, whereas the Platonic Good, as I understand it, is not mind-dependent. To conflate knowledge with "the Good" seems to be a conflation of two different realms, the epistemological and the ontological.

It seems a very strange idea to me, the "mean" being applied in such a way as to suggest that the perfect life is neither healthy nor sick. One supposes that the "mean" is inappropriately applied in such a case. Surely a perfect life would involve perfect health, among other things. To think of a perfect life as one in which one was neither healthy nor sick suggests a life in which one is not healthy but, on the other hand, is not badly off enough to count as actually sick. It suggests a dismal life of physiological and medical mediocrity.

A classical characterization of Socratic *elenchus* is that it can establish falsity, but not truth. For example, it can reveal inconsistencies, but it has no way to establish what the truth is. It seems to require, for its pragmatic value, an a s s u m p t i o n that the bulk of working morality is sound. It would be nice, of course, if this could be independently established, particularly as the morality of ancient Athens does not seem to be as that of other parts of the world and other times. If you knew that 'p' was true, and 'q' logically entailed '~ p', then you would be justified in rejecting 'q'. On the other hand, if one meets someone who thinks that 'p' is obviously false, he would not be likely to take 'q' 's incompatibility with 'p' as an argument against 'q'.

I have tried to make clear, at least from the point of view of analytic philosophy, a number of semantic and logistic issues which seem to be involved in your interesting project. In particular, I have given attention to certain problems of translation, formalization, and limitation of various logics with respect to such considerations. The safest thing to do, it seems to me, is to produce ordinary-language surrogates, or models, for the arguments involved, as you have done, thus resolving, hopefully not arbitrarily, the obscurities, or perhaps even careless opacities, of the Platonic text. Plato was a gifted poet, and probably would have been barred from his Republic, if it had ever been realized. Aristotle, pretty obviously, with his predicate logic, could have done a much better job of presenting clear arguments. Just because an argument looks simple, namely, it is short, does not mean that it is simple. Similarly, the standard symbolic logics, e.g. the prepositional calculus and, say, the first-order functional calculus, occasionally have a rather problematic relation to the logic of ordinary language. For example, 'All unicorns are blue' and 'All unicorns are not blue' would both count as true in "modern logic", given the non-existence of unicorns, but this would, at the least, be accounted peculiar in normal discourse. My colleague, Professor Alex Orenstein, has done some very interesting work in developing symbolic forms which are much closer to ordinary-language presuppositions, but this work, as yet, is on the whole unfamiliar to the logistic community. From my point of view, I think you would do better to propose model ordinarylanguage arguments to advance your project.

I am not commenting in any detail on the paper for a variety of reasons. For one thing, it is my surmise that it is now well on its way to a deserved publication. I congratulate you on what seems to me its depth and quality. To be sure, I am not a Platonic scholar, and my opinion is not of great interest or value in the complex, and to me somewhat arcane, area of Plato studies, a milieu in which you are obviously at home and where, as far as I can tell, you are thriving. Thank you very much for your interesting, impressive, carefully written and extensively documented article pertaining to Plato on Dialectic and Democracy, which I have enjoyed reading. It is a remarkable piece of well-argued scholarship. I found it informative, and I have little doubt I will find it useful as well. I am, of course, not a Plato scholar, but it seemed very good to me, and, knowing you, I am sure that it is objectively good.

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